1. Given that $i = 0.04$, ${}^{20}_{23}V_{15} = 0.585$, and ${}^{20}_{24}V_{15} = 0.600$, calculate p_{38}.

.979

2. Calculate ${}_{20}V_{45}$, given the following values:

(i) $P_{45} = .014$ (ii) $P^{\,1}_{45:\overline{20|}} = .022$ (iii) $P_{45:\overline{20|}} = .030$

~~.49557~~ .272727

3. Assume that mortality follows the Illustrative Life Table, with $i = 0.06$.
Also assume that deaths are uniformly distributed over each year of age.

Calculate $1000({}_5V^{(4)}_{35} - {}_5V_{35})$. .11727

4. A whole life insurance policy of 1000 is issued to a life age 35. The death benefit is payable at the moment of death. The policy is to be paid for by level quarterly premiums that are determined by the equivalence principle.

You are given:

i) $q_{35} = .003$

ii) $i = .06$

iii) $1000P^{(4)}(\bar{A}_{35}) = 7$

Assuming that deaths are uniformly distributed within each year of age, find the benefit reserve at time 1.

5. You are given:

i) $A_{x:\overline{n|}} = 0.25$

ii) $d = 0.03$

.96

Calculate ${}_{n-1}V_{x:\overline{n|}}$.

6. Calculate A_{45}, given:

i) ${}_{10}V_{25} = 0.1$

ii) ${}_{10}V_{35} = 0.2$

iii) $A_{25} = .15$

.388

7. You are given:

i) The force of mortality is a constant $\mu = 0.06$.

ii) The force of interest is $\delta = 0.04$.

Calculate $\Pr[{}_{10}L > 0 \mid T(50) > 10]$, where ${}_{10}L$ is the prospective loss random variable associated with a fully continuous whole life insurance.

.251189%

$n = 23.0258$

8. You are given $\delta = 0.05$, ${}_{10}p_{25} = 0.95$, $\bar{A}_{25:\overline{10}|} = 0.65$, and $\bar{a}_{25:\overline{30}|} = 15.0$

Calculate $\bar{a}_{35:\overline{20}|} + 100\ {}_{10}\bar{V}(\bar{A}_{25:\overline{30}|})$.

21.3242

9. Find ${}_{10}V_{30}$, given:

$$\ddot{a}_{30} = 20$$
$$P_{40} = .03$$
$$P_{50} = .05$$
$${}_{10}V_{40} + {}_{20}V_{30} = .625$$

10. For a fully continuous whole life insurance on (30), premiums are determined by the equivalence principle. Assume the following:

i) constant force of mortality and interest applies

ii) ${}^{2}\bar{A}_{30} = 0.25$

iii) $\bar{A}_{30} = 0.4$

0.25

Calculate $\mathrm{Var}[{}_5L \mid T(x) > 5]$, where ${}_5L$ is the prospective loss on the fully continuous whole life insurance issued to (30) at time 5.

ACT 3230 Actuarial Models II

Exam 2 Solutions – Chapter 7

February 13, 2008
4:00 p.m. – 5:15 p.m.

Instructor: Sheldon Liu, FSA, FCIA

1.

This deals with a 20-payment years, whole life insurance. We are past the premium payment period, so by the prospective method,
$^{20}_{23}V_{15} = A_{38} - 0$ and $^{20}_{24}V_{15} = A_{39} - 0$

$0.585 = {}^{20}_{23}V_{15} = A_{38} = vq_{38} + vp_{38}A_{39} = (1/1.04)[q_{38} + p_{38}(0.600)]$
$0.6084 = 1 - p_{38} + p_{38}(0.600) = 1 - 0.4p_{38}$
$p_{38} = (1 - 0.6084)/0.4 = 0.979$

2.

Recognize the 3-premium problem.
Using the retrospective method,

$_{20}V_{45:\overline{20|}} = P_{45:\overline{20|}}\,\ddot{s}_{45:\overline{20|}} - {}_{20}k_{45}$
$_{20}V_{45} = P_{45}\,\ddot{s}_{45:\overline{20|}} - {}_{20}k_{45}$
and $\ddot{s}_{45:\overline{20|}} = 1 / P_{45:\overset{\;\;1}{\overline{20|}}}$

So, $({}_{20}V_{45:\overline{20|}} - {}_{20}V_{45}) = (P_{45:\overline{20|}} - P_{45})\,\ddot{s}_{45:\overline{20|}}$
$1 - {}_{20}V_{45} = (.030 - .014) / .022$
$_{20}V_{45} = 0.2727$

<u>Alternative:</u>

$P^{1}_{45:\overline{20|}} = P_{45:\overline{20|}} - P_{45:\overset{\;\;1}{\overline{20|}}} = .030 - .022 = .008 = \dfrac{A^{1}_{45:\overline{20|}}}{\ddot{a}_{45:\overline{20|}}}$ (by Ch 6)

$.022 = P_{45:\overset{\;\;1}{\overline{20|}}} = \dfrac{{}_{20}E_{45}}{\ddot{a}_{45:\overline{20|}}}$

So, ${}_{20}k_{45} = \dfrac{A^{1}_{45:\overline{20|}}}{{}_{20}E_{45}} = \dfrac{P^{1}_{45:\overline{20|}}}{P_{45:\overset{\;\;1}{\overline{20|}}}} = .008 / .022 = 0.3636$

$$
\begin{aligned}
{}_{20}V_{45} &= P_{45}\ \ddot{s}_{45:\overline{20|}} - {}_{20}k_{45} \\
&= .014\,(1 / .022) - 0.3636 = 0.2727
\end{aligned}
$$

3.

$$
\begin{aligned}
{}_5V_{35} &= A_{40} - P_{35}\,\ddot{a}_{40} \\
&= A_{40} - (A_{35} / \ddot{a}_{35})\,\ddot{a}_{40} \\
&= 0.16132 - (0.12872 / 15.3926)\,(14.8166) \\
&= 0.037416777
\end{aligned}
$$

Under UDD-eya,

$${}_5V_{35}^{(4)} = (1 + \beta(4)\ P_{35}^{(4)})\ {}_5V_{35}$$

$$\ddot{a}_{35}^{(4)} = \alpha(4)\,\ddot{a}_{35} - \beta(4) = 1.00027(15.3926) - 0.38424 = 15.012516$$

$$P_{35}^{(4)} = A_{35} / \ddot{a}_{35}^{(4)} = 0.12872 / 15.012516 = 0.008574179$$

Therefore,

$$
\begin{aligned}
{}_5V_{35}^{(4)} &= (1 + \beta(4)\ P_{35}^{(4)})\ {}_5V_{35} \\
&= [1 + .38424(.008574179)](.037416777) \\
&= 0.037540048
\end{aligned}
$$

$$1000({}_5V_{35}^{(4)} - {}_5V_{35})$$
$$= 1000\,(0.037540048 - 0.037417) = 0.12327$$

4.

$$1000\ {}_1V^{(4)}(\overline{A}_{35}) = 1000P^{(4)}(\overline{A}_{35})(\ddot{a}_{35:\overline{1|}}^{(4)} / {}_1E_{35}) - 1000(\overline{A}^{\,1}_{35:\overline{1|}} / {}_1E_{35})$$

$${}_1E_{35} = vp_{35} = .997 / 1.06 = 0.940566$$

Under UDD-eya,

$$
\begin{aligned}
\overline{A}^{\,1}_{35:\overline{1|}} &= (i / \delta)\,A^{\,1}_{35:\overline{1|}} = (.06 / \ln(1.06))\ vq_{35} = (.06 / \ln(1.06))*(.003 / 1.06) \\
&= 0.002914
\end{aligned}
$$

$$
\begin{aligned}
A^{(4)}_{35:\overline{1|}} &= (\delta / i^{(4)})\ \overline{A}^{\,1}_{35:\overline{1|}} + A_{35:\overline{1|}}^{\ \ \,1} \\
&= \ln(1.06)/(0.05870)\ vq_{35} + vp_{35} \\
&= \ln(1.06)/(0.05870)\,(.003 / 1.06) + (.997 / 1.06) \\
&= 0.943375
\end{aligned}
$$

where $(1 + i) = (1 + i^{(4)}/4)^4 \rightarrow i^{(4)} = 0.05870$

$\ddot{a}^{(4)}_{35:\overline{1}|} = (1 - A^{(4)}_{35:\overline{1}|}) / d^{(4)} = (1 - 0.943375) / 0.05785 = 0.9788169$

Therefore,

$1000\ {}_1V^{(4)}(\bar{A}_{35}) = 1000P^{(4)}(\bar{A}_{35})(\ddot{a}^{(4)}_{35:\overline{1}|} / {}_1E_{35}) - 1000(\bar{A}^{\ 1}_{35:\overline{1}|} / {}_1E_{35})$

$= 7 * [0.9788169 / 0.940566] - 1000 * [0.002914 / 0.940566]$

$= 4.186541$

5.

$$P_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} = \frac{dA_{x:\overline{n}|}}{1 - A_{x:\overline{n}|}} = (.03 * .25)/(1 - .25) = 0.01$$

$${}_{n-1}V_{x:\overline{n}|} = A_{x+n-1:\overline{1}|} - P_{x:\overline{n}|}\ \ddot{a}_{x+n-1:\overline{1}|}$$

$= v - P_{x:\overline{n}|}$ (since for any age y, $A_{y:\overline{1}|} = v$ and $\ddot{a}_{y:\overline{1}|} = 1$)

$= 1 - d - P_{x:\overline{n}|}$

$= 1 - 0.03 - 0.01$

$= 0.96$

6.

Using the annuity form of the whole life reserve, we get

$${}_{10}V_{25} = 1 - \frac{\ddot{a}_{35}}{\ddot{a}_{25}} \rightarrow \frac{\ddot{a}_{35}}{\ddot{a}_{25}} = 1 - 0.1 = 0.9$$

$${}_{10}V_{35} = 1 - \frac{\ddot{a}_{45}}{\ddot{a}_{35}} \rightarrow \frac{\ddot{a}_{45}}{\ddot{a}_{35}} = 1 - 0.2 = 0.8$$

So, ${}_{20}V_{25} = 1 - \frac{\ddot{a}_{45}}{\ddot{a}_{25}} = 1 - (\frac{\ddot{a}_{35}}{\ddot{a}_{25}})(\frac{\ddot{a}_{45}}{\ddot{a}_{35}}) = 1 - (.9)(.8) = .28$

$${}_{20}V_{25} = \frac{A_{45} - A_{25}}{1 - A_{25}} \rightarrow A_{45} = ({}_{20}V_{25})(1 - A_{25}) + A_{25} = (.28)(1 - .15) + .15 = 0.388$$

7.

$$F_{{}_tL}(y) = 1 - F_{T(x+t)}\left(-\frac{1}{\delta}\log\left[\frac{\delta y + \overline{P}(\overline{A}_x)}{\delta + \overline{P}(\overline{A}_x)}\right]\right)$$

$$\Pr[{}_tL > 0 | T(x) > t] = 1 - F_{{}_tL}(0) = F_{T(x+t)}\left(-\frac{1}{\delta}\log\left[\frac{\mu}{\delta + \mu}\right]\right)$$

$T(x+t)$ follows an exponential distribution since the force of mortality is constant. Thus,

$$F_{T(x+t)}(s) = 1 - e^{-\mu s}$$

$$\Pr[{}_tL > 0 | T(x) > t] = 1 - e^{(\mu/\delta)\log(\frac{\mu}{\delta+\mu})}$$

$$= 1 - (0.6)^{0.06/0.04} = 0.53524$$

8.

$${}_{10}E_{25} = v^{10}\ {}_{10}p_{25} = e^{-.05(10)}(.95) = 0.576204$$

$$\overline{a}_{25:\overline{10|}} = (1 - \overline{A}_{25:\overline{10|}})/\delta = (1 - .65)/.05 = 7$$

$$\overline{a}_{35:\overline{20|}} = (\overline{a}_{25:\overline{30|}} - \overline{a}_{25:\overline{10|}})/{}_{10}E_{25} = (15 - 7)/.576204 = 13.883969$$

$$\overline{A}_{25:\overline{30|}} = 1 - \delta * \overline{a}_{25:\overline{30|}} = 1 - 0.05(15.0) = 0.25$$

$$\overline{A}_{35:\overline{20|}} = 1 - \delta * \overline{a}_{35:\overline{20|}} = 1 - 0.05(13.883969) = 0.305802$$

$$\overline{P}(\overline{A}_{25:\overline{30|}}) = \overline{A}_{25:\overline{30|}} / \overline{a}_{25:\overline{30|}} = 0.25/15 = 0.016666$$

$${}_{10}\overline{V}(\overline{A}_{25:\overline{30|}}) = \overline{A}_{35:\overline{20|}} - \overline{P}(\overline{A}_{25:\overline{30|}})\,\overline{a}_{35:\overline{20|}}$$

$$= 0.305802 - (0.016666)(13.883969) = 0.074402$$

Therefore, $\overline{a}_{35:\overline{20|}} + 100\ {}_{10}\overline{V}(\overline{A}_{25:\overline{30|}}) = 13.883969 + 100(0.074402) = 21.324178$

<u>Alternative:</u>

$${}_{10}E_{25} = v^{10}\ {}_{10}p_{25} = e^{-.05(10)}(.95) = 0.576204$$

$$\overline{a}_{25:\overline{10|}} = (1 - \overline{A}_{25:\overline{10|}})/\delta = (1 - .65)/.05 = 7$$

$$\overline{s}_{25:\overline{10|}} = \overline{a}_{25:\overline{10|}}/{}_{10}E_{25} = 7/0.576204 = 12.148473$$

$$\overline{A}_{25:\overline{30|}} = 1 - \delta * \overline{a}_{25:\overline{30|}} = 1 - 0.05(15.0) = 0.25$$

$$\overline{P}(\overline{A}_{25:\overline{30|}}) = \overline{A}_{25:\overline{30|}} / \overline{a}_{25:\overline{30|}} = 0.25/15 = 0.016666$$

$$\overline{A}^{\,1}_{25:\overline{10|}} = \overline{A}_{25:\overline{10|}} - \overline{A}_{25:\overline{10|}}^{\ \ \ \ \ 1} = 0.65 - 0.576204 = 0.073796$$

$$_{10}k_{25} = \bar{A}^{\,1}_{25:\overline{10|}} \,/\, _{10}E_{25} = 0.073796 / 0.576204 = 0.128072$$

$$_{10}\bar{V}(\bar{A}_{25:\overline{30|}}) = \bar{P}(\bar{A}_{25:\overline{30|}})\,\bar{s}_{25:\overline{10|}} - {}_{10}k_{25} = (0.016666)(12.148473) - 0.128072$$
$$= 0.074402$$

$$_{10}\bar{V}(\bar{A}_{25:\overline{30|}}) = \bar{A}_{35:\overline{20|}} - \bar{P}(\bar{A}_{25:\overline{30|}})\,\bar{a}_{35:\overline{20|}}$$
$$_{10}\bar{V}(\bar{A}_{25:\overline{30|}}) = 1 - \delta * \bar{a}_{35:\overline{20|}} - \bar{P}(\bar{A}_{25:\overline{30|}})\,\bar{a}_{35:\overline{20|}}$$
$$\bar{a}_{35:\overline{20|}} = \frac{1 - {}_{10}\bar{V}(\bar{A}_{25:\overline{30|}})}{\delta + \bar{P}(\bar{A}_{25:\overline{30|}})} = (1 - 0.074402) / (0.05 + 0.016666) = 13.883969$$

Or, $_{10}\bar{V}(\bar{A}_{25:\overline{30|}}) = 1 - \dfrac{\bar{a}_{35:\overline{20|}}}{\bar{a}_{25:\overline{30|}}} \rightarrow \bar{a}_{35:\overline{20|}} = [1 - {}_{10}\bar{V}(\bar{A}_{25:\overline{30|}})]\,\bar{a}_{25:\overline{30|}}$

So, $\bar{a}_{35:\overline{20|}} = (1 - 0.074402)(15) = 13.883969$

Therefore, $\bar{a}_{35:\overline{20|}} + 100\ {}_{10}\bar{V}(\bar{A}_{25:\overline{30|}}) = 13.883969 + 100(0.074402) = 21.324178$

9.

$$_{10}V_{40} = \frac{P_{50} - P_{40}}{P_{50} + d} = \frac{.05 - .03}{.05 + d} = \frac{.02}{.05 + d}$$
$$P_{30} = 1 / \ddot{a}_{30} - d = 1/20 - d = .05 - d$$
$$_{20}V_{30} = \frac{P_{50} - P_{30}}{P_{50} + d} = \frac{.05 - (.05 - d)}{.05 + d} = \frac{d}{.05 + d}$$

$$_{10}V_{40} + {}_{20}V_{30} = \frac{.02}{.05 + d} + \frac{d}{.05 + d} = \frac{.02 + d}{.05 + d} = 0.625$$

$\rightarrow$ $.02 + d = .625(.05 + d)$

$.02 + d = .03125 + .625d$

$.375d = .01125$

$d = 0.03$

So, $P_{30} = .05 - d = .05 - .03 = .02$

$$_{10}V_{30} = \frac{P_{40} - P_{30}}{P_{40} + d} = \frac{.03 - .02}{.03 + .03} = \frac{.01}{.06} = 1/6 \text{ or } .1667$$

10.

Under CF,

$$^2\bar{A}_x = \frac{\mu}{\mu+2\delta} \quad \text{and} \quad \bar{A}_x = \frac{\mu}{\mu+\delta} \quad \text{for all } x.$$

$$\text{Var}[{}_5L \mid T(x) > 5] = [1 + \bar{P}(\bar{A}_x)/\delta]^2 * [{}^2\bar{A}_{x+5} - (\bar{A}_{x+5})^2]$$

$$= \frac{{}^2\bar{A}_{x+5} - (\bar{A}_{x+5})^2}{(1-\bar{A}_x)^2} = \frac{.25-(.4)^2}{(1-.4)^2} = .09 / .36 = 0.25$$

This question was much easier than intended. You should note that the insurance terms in the numerator are at time 5, while the premium term still uses time 0.